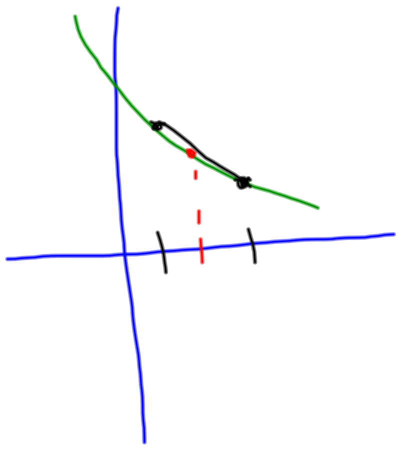
Barron's 91

symmetric difference quotient

$$SDQ(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

for $f(x) = 5^x$, what is the estimate of $f'(2)$ obtained by using the SDQ with $h = 0.03$?



$$SDQ(2) = \frac{f(2+0.03) - f(2-0.03)}{2(0.03)}$$

$$\approx 40.252$$

92) To how many places is the SDQ accurate when it is used to approximate $f'(0)$ for $f(x)=4^x$ and $h=0.08$

$$SDQ(x=0) = \frac{4^{0+0.08} - 4^{0-0.08}}{2(0.08)}$$

$$= \underline{1.38}9$$

$$(e^{\ln 4})^x$$

$$e^{x\ln 4}$$

$$\frac{d}{dx}(e^{x\ln 4})$$

$$= e^{x\ln 4} \cdot \underline{\ln 4}$$

$$\frac{d}{dx}(4^x) = (\ln 4)4^x$$

$$f'(0) = (\ln 4)4^0 = \ln 4 = \underline{1.38}6$$

If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

$f(h(x)) = x$

$f'(h(x)) \cdot h'(x) = 1$ so $h'(x) = \frac{1}{f'(h(x))}$

$h'(3) = \frac{1}{f'(h(3))}$

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2} = \frac{-1}{x^2}$

$h(3) = x$

if $\frac{1}{x} = 3$

so $h(3) = \frac{1}{3}$

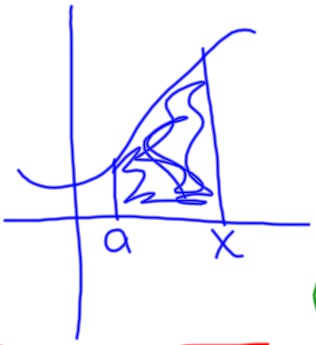
$h'(3) = \frac{1}{\frac{-1}{\left(\frac{1}{3}\right)^2}} = -\frac{1}{9}$

Jan 4-7:28 AM

6.6 Fundamental Theorem of Calculus (part 1)

Recall in 6.1 we looked at an area function

$A(x)$ = area under $f(x)$ from a value of "a" to some unspecified x.



we observed that $A'(x) = f(x)$.
This is always true.
So $A(x)$ is an antiderivative of $f(x)$.

$A(a) = 0$.

$A(x) = \int f(x)\,dx = F(x) + C$

we also know that the area under $f(x)$ between a and a known value b is $\int_a^b f(x)\,dx$.

So we have....

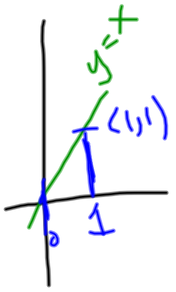
$$\text{Area under } f \text{ over } [a,b] = \int_a^b f(x)\,dx = A(b) - A(a)$$
$$= (F(b) + C) - (F(a) + C)$$
$$= F(b) - F(a)$$

where $F(x)$ is ANY antiderivative.

FTC(1)

$$\int_a^b f(x)\,dx = F(b)-F(a)$$ where $F(x)$ is an antiderivative of $f(x)$

$$\int_0^1 x\,dx =$$



$$= \frac{1}{2} = F(1)-F(0)$$

$$= \frac{x^2}{2}\Big|_{x=1} - \frac{x^2}{2}\Big|_{x=0}$$

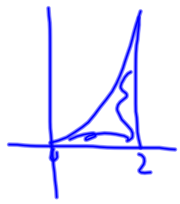
$$= \frac{1}{2} - 0 = \frac{1}{2}$$

$$\star = \left(\frac{x^2}{2}+4\right)\Big|_{x=1} - \left(\frac{x^2}{2}+4\right)\Big|_{x=0}$$

$$4\tfrac{1}{2} - 4 = \frac{1}{2}$$

calculate the area under $y=x^2$ from 0 to 2.

$$\text{Area} = \int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3} - \frac{0}{3} = \frac{8}{3}$$



calculate the area under $y = \sin x$ from 0 to π.

$$\text{Area} = \int_0^{\pi} \sin x\,dx = (-\cos x)\Big|_0^{\pi}$$

$$= -\cos\pi - (-\cos 0) = -(-1)-(-1)$$

$$= 2$$